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Assignment 9: MTH 213, Fall 2017

Ayman Badawi

Mueez Khan

QUESTION 1. Given string1, say S_1 : 0 1 0 1 1 0 and string2, say S_2 : 1 1 1 0 0 1
a) Find $S_1 \wedge S_2$
b) Find $S_1 \vee S_2$
c) Find $S_1 \oplus S_2$
d) Find $\neg S_1 \vee S_2$

QUESTION 2. Convince me that $S_1 \wedge (S_2 \rightarrow S_3) \equiv (S_1 \wedge \neg S_2) \vee (S_1 \wedge S_3)$

QUESTION 3. Given a sequence $\{b_n\}_{n=0}^{\infty}$, where $b_0 = 3, b_1 = 3$, and $b_n = 4b_{n-1} - 4b_{n-2} + 2^n + 1$. Find a general formula for b_n. **Solution: We only need to take care of the linear recurrence (undetermined part), other part $2^n + 1$ is determined. So as usual**

$$\alpha^n = 4\alpha^{n-1} - 4\alpha^{n-2} \ \ (\alpha \neq 0), \quad then\ divide\ both\ equations\ by\ \alpha^{n-2}, we\ get$$
$$\alpha^2 - 4\alpha + 4 = 0,\ Hence\ C_b(\alpha) = \alpha^2 - 4\alpha + 4$$
$$set\ \ C_b(\alpha) = \alpha^2 - 4\alpha + 4 = 0,\ we\ get\ \alpha = 2\ \alpha = 2$$
$$Hence\ b_n = c_1(2^n) + c_2 n(2^n) + 2^n + 1$$
$$now\ 3 = b_0 = c_1 + 0 + 1 + 1,\ so\ c_1 = 1.\ Also\ \ 3 = b_1 = 2(2^1) + c_2(1)(2^1) + 2^1 + 1, so\ \ c_2 = -1$$

$$b_n = 2^n - n(2^n) + 2^n + 1 = 2(2^n) - n(2^n) + 1 = 2^{(n+1)} - n(2^n) + 1$$

QUESTION 4. Given a sequence $\{b_n\}_{n=0}^{\infty}$, where $b_0 = 1, b_1 = 12$, and $b_n = b_{n-1} + 6b_{n-2}$. Find a general formula for b_n.

QUESTION 5. Given a sequence $\{b_n\}_{n=0}^{\infty}$, where $b_0 = 1, b_1 = 12$, and $b_n = b_{n-1} + 6b_{n-2} + n^2 - n + 31$. Find a general formula for b_n.

QUESTION 6. Write down T or F

(i) If $\exists x \in R$ such that $x + 4 = 5$, then $x^2 + 2 = 4$

(ii) If $\exists x \in R$ such that $x^2 + 4 = 5$, then $x + 2 = 3$

(iii) If $\exists x \in N$ such that $x^2 + 4 = 5$, then $x + 2 = 3$

(iv) If $\exists x < 0$ such that $x^2 + 4 = 5$, then $x^3 + 2 = 1$

(v) If $\exists x \in R$ such that $x^2 + 1 = -5$, then $x^3 + 2x - e^x = -34$

(vi) If $\exists x \in R$ such that $x^2 + 1 = -5$, then $x^3 + \sqrt{x} + ln(4x) + 7x^2 = 10^{213}4$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

Assignment 9

Q1) S_1 : 0 1 0 1 1 0 $\qquad \sim S_1$: 1 0 1 0 0 1

S_2 : 1 1 1 0 0 1

a) $S_1 \wedge S_2$: 0 1 0 0 0 0

b) $S_1 \vee S_2$: 1 1 1 1 1 1

c) $S_1 \oplus S_2$: 1 0 1 1 1 1

d) $\sim S_1 \vee S_2$: 1 1 1 0 0 1

Q2) $S_1 \wedge (S_2 \rightarrow S_3) \equiv (S_1 \wedge \sim S_2) \vee (S_1 \wedge S_3)$

S_1	S_2	S_3	$S_1 \wedge \sim S_2$	$S_1 \wedge S_3$	$S_2 \rightarrow S_3$	$(S_1 \wedge \sim S_2) \vee (S_1 \wedge S_3)$	$S_1 \wedge (S_2 \rightarrow S_3)$
0	0	0	0	0	1	0	0
0	0	1	0	0	1	0	0
0	1	0	0	0	0	0	0
0	1	1	0	0	1	0	0
1	0	0	1	0	1	1	1
1	0	1	1	1	1	1	1
1	1	0	0	0	0	0	0
1	1	1	0	1	1	1	1

same

$\Rightarrow S_1 \wedge (S_2 \rightarrow S_3) \equiv (S_1 \wedge \sim S_2) \vee (S_1 \wedge S_2)$

Q3) $\{b_n\}_{n=0}^{\infty}$ $b_0 = 3$ $b_1 = 3$

$b_n = 4b_{n-1} - 4b_{n-2} + 2^n + 1$

$$C_b(\alpha) \Rightarrow \frac{\alpha^n}{\alpha^{n-2}} = \frac{4\alpha^{n-1}}{\alpha^{n-2}} - \frac{4\alpha^{n-2}}{\alpha^{n-2}}$$

$$\rightarrow \alpha^2 = 4\alpha - 4$$

$$\alpha^2 - 4\alpha + 4 = 0 \qquad C_b(\alpha) = \alpha - 4\alpha + 4$$

$$\underbrace{\alpha = 2 \qquad \alpha = 2}_{\text{same roots}}$$

$$b_n = C_1(2^n) + C_2\, n(2^n) + 2^n + 1$$

$$b_0 = 3 = C_1 + 2 \rightarrow \underline{C_1 = 1}$$ substitute c_1

$$b_1 = 3 = 2C_1 + 2C_2 + 3$$

$$3 = 2 + 2C_2 + 3$$

$$2C_2 = -5 + 3$$

$$\underline{C_2 = -1}$$

$$b_n = 2^n - n2^n + 2^n + 1$$

$$b_n = 2(2^n) - n2^n + 1$$

$$b_n = 2^{n+1} - n2^n + 1$$

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Q4) $\{b_n\}_{n=0}^{\infty}$ $b_0 = 1$ $b_1 = 12$

$b_n = b_{n-1} + 6b_{n-2}$

$$\frac{\alpha^n}{\alpha^{n-2}} = \frac{\alpha^{n-1}}{\alpha^{n-2}} + \frac{6\alpha^{n-2}}{\alpha^{n-2}}$$

$$\rightarrow \alpha^2 = \alpha + 6$$

$$\alpha^2 - \alpha - 6 = 0 \qquad C_b(\alpha) = \alpha^2 - \alpha - 6$$

$$\alpha = 3 \qquad \alpha = -2$$

Note... (-2)^n = - (2)^n only if n is odd
So b_n = c_1(3)^n + c_2(-2)^n
so b_0 = 1 = c_1 + c_2
b_1 = 12 = 3c_1 - 2c_2
Hence c_1 = 14/5, c_2 = -9/5
b_n = (14/5)(3)^n + (-9/5)(-2)^n

Q 5) $\{b_n\}_{n=0}^{\infty}$ $b_0 = 1$ $b_1 = 12$

$b_n = b_{n-1} + 6b_{n-2} + n^2 - n + 31$

$$\frac{\alpha^n}{\alpha^{n-2}} = \frac{\alpha^{n-1}}{\alpha^{n-2}} + \frac{6\alpha^{n-2}}{\alpha^{n-2}}$$

$\alpha^2 = \alpha + 6$

$\alpha^2 - \alpha - 6 = 0$ $C_b(\alpha) = \alpha^2 - \alpha - 6$

$\alpha = 3$ $\alpha = -2$

So b_n = c_1(3)^n + c_2(-2)^n + n^2 - n + 31

so b_0 = 1 = c_1 + c_2 + 31 implies -30 = c_1 + c_2 (1)

b_1 = 12 = 3c_1 - 2c_2 + 1 - 1 + 31 implies -19 = 3c_1 - 2c_2 (2)

Solve (1) and (2)

Hence c_1 = -79/5, c_2 = -71/5

b_n = (-79/5)(3)^n + (-71/5)(-2)^n + n^2 - n + 31

Q6) i) If $\underbrace{\exists x \in \mathbb{R} \text{ such that } x+4=5}_{\hookrightarrow T \ (x=1)}$, then $\underbrace{x^2+2=4}_{\hookrightarrow F}$.

Answer = F

ii) If $\underbrace{\exists x \in \mathbb{R} \text{ such that } x^2+4=5}_{\hookrightarrow T \ (x=1)}$, then $\underbrace{x+2=3}_{\hookrightarrow T}$.

Answer = F

x = -1 and -1 in R and x^2 + 4 = 5, but X + 2 not = 3. Hence conclusion is F. So the whole statement is F. To make it true I should add.. then x + 2 = 3 OR x + 2 = 1

iii) If $\underbrace{\exists x \in N \text{ such that } x^2+4=5}_{\hookrightarrow T \ (x=1)}$, then $\underbrace{x+2=3}_{\hookrightarrow T}$

Answer = T

iv) If $\underbrace{\exists x < 0 \text{ such that } x^2+4=5}_{\hookrightarrow T \ (x=-1)}$, then $\underbrace{x^3+2=1}_{\hookrightarrow T}$

Answer = T

v) If $\underbrace{\exists x \in \mathbb{R} \text{ such that } x^2+1=-5}_{\hookrightarrow F}$, then $x^3+2x-e^x=-34$

Answer = T

vi) If $\underbrace{\exists x \in \mathbb{R} \text{ such that } x^2+1=-5}_{\hookrightarrow F}$

then $x^3+\sqrt{x}+\ln(4x)+7x^2=10^{213}4$

Answer = T

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